Veracity Logic

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- Try to pin down and then explore in a logical setting what this means
- Attempt to formalise as much of veracity as we can in order to understand the way it works better

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- These cases in some sense wear their veracity on their sleeve: it is immediate, there is "a piece of veracity", so this is the *atomic veracity*.
- It cannot be further analysed in terms of asking whose hands it has passed through, how it has be modified or added to since none of this has ever happened to it.

Atomic veracity

- The central idea: Veracity is denoted by a *witness* to the act of association within a claim. It is this that is objectified and then tracked. This track will be what is looked to when it is said “how is it known that this bar code correctly identifies this object?”

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- There is a special veracity claim $\perp$ which has no witnesses, i.e. it is the claim that is never upheld, can never be upheld.

Rules for veracity

- This leads to the first proof rule:

$$\frac{a \in \bot}{a \in A}\,\bot^-$$

This rule says that if, in the course of reasoning, somehow it is shown that the claim that can never have veracity does in fact have it, then it can be shown that *any claim* has veracity. Call this rule $\bot^-$ for "$\bot$ elimination".

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- Other rules, amongst many, would be:

$$\frac{a \in A \quad b \in B}{(a, b) \in A \wedge B}\ \wedge^{+}$$

$$\frac{(a, b) \in A \wedge B}{a \in A}\ \wedge^{-1}$$

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- What does saying "there are claims A and B and it is known that A has a witness even if nothing is known about B, so it is known that one or the other has one: that is, a claim of A or B is witnessed"?
- Might choose to formalise this by saying

$$\frac{a \in A}{a \in A \vee B}$$

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- The conclusion does not record which of the alternatives have been relied on to reach it; does the claim of one or the other rest on the fact that the first was witnessed, or the second?
- Righting this means doing something like

$$\frac{a \in A}{i(a) \in A \vee B} \vee^{+}1$$

$$\frac{b \in B}{j(b) \in A \vee B} \vee^{+}2$$

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- Imagine that by assuming that claim A has veracity, i.e. that the judgement $x \in A$ has been shown for some arbitrary witness x, can show that claim B has veracity, i.e. can show $b(x) \in B$. Here $b(x)$ is just notation for a term which may contain x free.

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- Thinking about a typical logic, introduce an *implication claim* to reflect this, i.e. to discharge the assumption, so the claim becomes

$$A \implies B$$

  but what would a witness to *this* claim plausibly look like?

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- Then define negation in terms of $\perp$ so that $\neg A$ is $A \implies \perp$. A witness to a claim of $\neg A$ takes a witness to A and give a witness to $\perp$. But $\perp$ has no witness, so a witness to $\neg A$ is not possible, as expected by the informal understanding of saying a claim has no witnesses

Where are we heading?

- This requirement that to justify a disjunction of claims it has to be demonstrated which of the claims were justified before (which is the role that the tags on the witnesses are playing in the rules) means that, for example, the claim $A \vee \neg A$ is also not justifiable without saying which claim is witnessed

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- $A \vee \neg A$ doesn't survive the question: yes, but can you show, whatever A is, the witness that assures the veracity of the claim here?
- And the view that witnesses to implications are functions leads us in the same direction...
- ....to the thought that this is reinterpreting intuitionistic logic

Trust, and people or other actors

- The argument is that the logic work above covers the verifiability (checking a proof is easy) and truth aspects of veracity. What is not yet settled is the trust aspect (the authenticity is left for now—yet to have any ideas on how it might be treated, or even what it is)

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- Add the notion of an *actor* from a set Act to our witnesses, so write $a^k \in A$ to show that actor $k \in Act$ accepts witness a as demonstrating claim A
- For a single actor, all of the logical rules so far are unchanged...just add the actor superscript

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- Write kTl to say that $k, l \in Act$ are in a trust relationship where k trusts l
- Add a rule so that trust and the other aspects of veracity already covered can interact

$$\frac{kTl \quad a^l \in A}{a^k \in A}\, trust\ \ T$$

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- Certainly reflexive, and certainly not symmetric

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- We write

$$a^k_{0.5} \in A$$

for k believes with strength 0.5 that A supports the claim A (and we drop the subscript in the case it's 1.0).

- Then the apparent transitivity above only works if $kT_{1.0}l$ and $lT_{1.0}m$, i.e. k trusts l completely, and the same for l and m, and that makes the apparent transitivity look reasonable

Trust, and people or other actors

- We recast the *trust T* rule as

$$\frac{kT_x l \quad a^l_y \in A}{a^k_{x.y} \in A}\ \textit{trust}\ T$$

- If instead $kT_{0.5}l$ and $lT_{0.4}m$ then I'd say $kT_{0.2}m$ and the proof above supports this, rewritten as

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- i.e. if $a^m_{1.0} \in A$ and $kT_{0.5}l$ and $lT_{0.4}m$ then $a^k_{0.2} \in A$

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- What happens if we have more than one trail of verification for a claim? Important for handling veracity during disputed claims